3-5-man mutual zugzwangs in chess

3-5 MAN MUTUAL ZUGZWANGS IN CHESS

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ABSTRACT

This note reports the work of Wirth and Karrer in twin-sourcing all mutual zugzwang positions, *mzugs*, in 2-5-man endgames. This paper tabulates the mzug statistical data, gives examples of maximal mzugs and refers to a chess endgame website where further data is to be found.

1. INTRODUCTION

Wirth and Nalimov have generated all 2-to-5-man endgame tables (EGTs) to the Depth-to-Conversion (DTC) and Depth-to-Mate (DTM) metrics respectively (Wirth and Nievergelt, 1999; Nalimov, Haworth and Heinz, 2000). Wirth and Karrer scanned the DTC and DTM EGTs respectively, second-sourcing all the mzugs.

A reciprocal or mutual zugzwang, *mzug*, in chess is a position where, ironically, each side could get a better result in theory if it were the other side's turn to move. There are three types of mzug:

ww	' = /1-0'	a 'White win' mzug ... the position is a wtm draw and a btm win for White
bw	'0-1/ = '	a 'Black win' mzug ... the position is a wtm win for Black and a btm draw
fp	'0-1/1-0'	a 'full point' mzug ... the side that has to move loses.

They are relatively rare and a running theme in Nunn's endgame trilogy (1992, 1994, 1995). Many endgame studies exploit the mzug theme at some point (Roycroft, 1972; Beasley, 1996; Elkies, 1998). Many counts of mzugs, principally by Rasmussen (1991-2000), have already been published: they confirm and are confirmed by the data here. This article completes the coverage of the 2-to-5-man domain, providing DTC and DTM data and examples of maximal mzugs in these two metrics.

2. THE RESULTS

Karrer (2000) scanned Nalimov's 3-to-5-man EGTs (Hyatt, 2001; Tamplin, 2001b) for mzug positions, giving:

- a list of mzugs together with statistics about counts and DTM-maximal depths
  n.b. for a full-point mzug, the *depth* is taken to be the sum of the wtm and btm depths
- a list of the DTM-maximal mzugs

The lists were then passed to workers in this field including Elkies, Rasmussen, Roycroft, Tamplin and Wirth. Wirth then computed the 4-1 EGTs to complete his suite of 2-to-5-man EGTs and executed the analogous mzug scan on them. Haworth collated the resulting statistics, confirmed full agreement between the results of Karrer and Wirth, and with the DTM data for the EGTs which Rasmussen (2000) had computed. He also identified mzugs which were maximal in terns of both the DTC and DTM metrics.

Tamplin (2001a) confirmed that the scans of DTC and DTM tables had yielded exactly the same sets of mzugs of each type, valuable confirmation in itself of the integrity of the EGTs. There are 21,677 *ww*, 3,395 *bw* and 33 *fp* 2-to-5-man-mzugs. These occur in 59 of the 146 endgames. The full point mzugs occur in just six endgames, namely KBPKP, KNPKP, KPKP, KPPKP, KPPKR and KRPKP: each features at least two Pawns. The complete data, statistics and various sets of positions, are available (Tamplin, 2001b). Here, the statistics are in Table 1 and the examples of maximal mzugs are in Table 2. Some explanatory notes are appropriate:

- White has at least as many men as Black: the men are listed in the standard K-Q-R-B-N-P order and endgames are listed in alphabetical order. The GBR code *qrcn.wb* indicates an endgame with *w* white and *b* black Pawns, *q mod 3* white and $\lfloor q/3 \rfloor$ black Queens. *r*, *c* and *n* similarly define the Rs, Bs and Ns.

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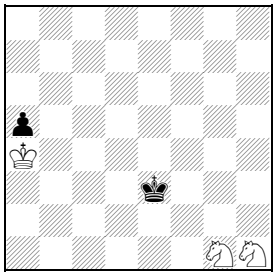
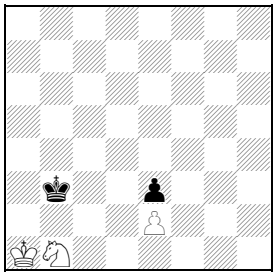
- with one exception, all positions are essentially unique, i.e. they cannot be transformed into another listed position by board transformation or by switching colours. The exception is that in symmetric endings, only KPKP here, the set of *bw* mzugs is acknowledged even though it is transformed by colour-reversal into the set of *ww* mzugs. The brackets in Table 1 highlight this equivalence.
- depths are in side-to-move moves. In Table 2, *c*, *m* and *cm* denote a DTC-, DTM- and DTC-_&_DTM-maximal position respectively. *ww*, *bw* and *fp* denote the three types of mzug as above.
- the positions are in *canonical form* in the sense that:

  the wK is confined to a-d for endgames with Pawns, and to a1-d1-d4 for endgames without Pawns
  if there are no Pawns and the wK is on a1-d4, the bK is confined to a1-h1-h8
  if both Kings are on a1-h8, only one of probably two equivalent positions is included in the count.
- a/the DTC-maximal mzug with maximal DTM depth has been cited. A DTM-maximal mzug with maximal DTC depth has also been cited if different, as it is in 16 endgames.
- the following 55 2-to-4-man and 3-2 endgames have no mzugs:

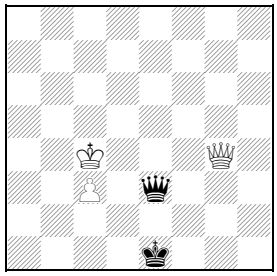
  KBBK, KBBKB, KBK, KBKB/N, KBNK, KK, KNK, KNKN, KNNK, KNNKB, KQBK, KQBKB/N/P/R, KQK, KQKB/N/P/Q/R, KQNK, KQNKB/N/P/R, KQPK, KQPB/N/P, KQQK, KQQKB/N/P/Q/R, KQRK, KQRKB/N/P/R, KRBK, KRBKB/N, KRK, KRKR, KRNK, KRNKB/P, KRPK, KRRK, KRRKN/P/R.
- the only 4-1 endgames with mzugs are KBPPK, KNPPK and KPPPK, echoing the fact that the only 3-1 endgames with mzugs are KBPK, KNPK and KPPK.



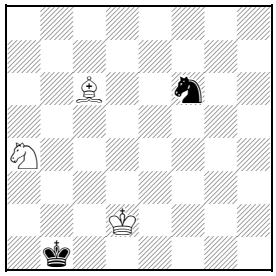
MZ1. The deepest mzug



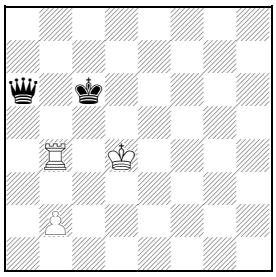
MZ2. The deepest *fp* mzug



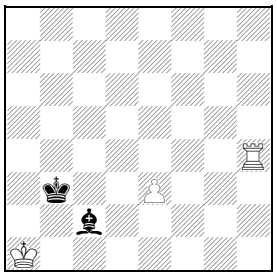
MZ3. btm, *dc* = 91



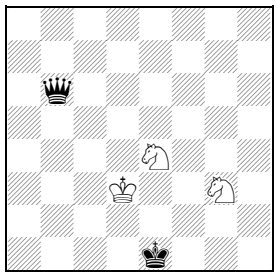
MZ4. btm, *dc* = 67



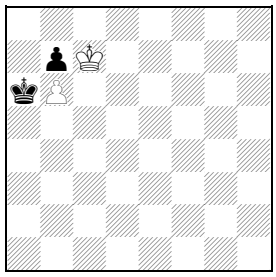
MZ5. wtm, *dc* = 63



MZ6. btm, *dc* = 58



MZ7. wtm, *dc* = 53



MZ8. Le Trébuchet

3. HIGHLIGHTS

KNNKP	MZ1	max 3-to-5-man mzug: *dc* = *dm* = 105 moves.
KNPKP	MZ2	max *fp* mzug: (wtm) *dc* = 1 and *dm* = 17, (btm) *dc* = 13 and *dm* = 28.
KQPKQ	MZ3	max KQPKQ mzug: *dc* = 91 and *dm* = 102.
KBNKN	MZ4	max KBNKN mzug: *dc* = 67 and *dm* = 97.
KBNKQ		8/8/q7/8/3K4/2N5/8/k1B5: only maximal P-less mzug with both Kings on a1-h8.
KPPKR		only fp mzugs not having Pawns on both sides: one is maxDTC, the other maxDTM.
KPKP	MZ8	the max fp mzug: like most KPKP positions, it has a twin with colours exchanged.
KPKP		provides 15 of the 33 full-point mzugs, all of the well known Le Trébuchet type.
KRKN		8/8/8/4k3/3R4/2K5/1n6/8: diagonally symmetric mzug.

All full-point mzugs feature at least two pawns: a conjecture is that there is a 6-man fpz with only one pawn. The sets of maxDTC mzugs and maxDTM mzugs are either disjoint, identical or one is contained within the other: further details are given below.

					—— DTC-maximal mzugs ——									—— DTM-maximal mzugs ——									ZC ^ ZM		
Endgame		mzug count			counts			max DTC			max-DTC mzugs' max DTM			counts			max DTM			max-DTM mzugs' max-DTC			counts		
Title	w-b	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp
KBBKN	3-2	1	0	0	1	0	0	2	---	---	2	---	---	1	0	0	2	---	---	2	---	---	1	0	0
KBBKP	3-2	1	0	0	1	0	0	2	---	---	2	---	---	1	0	0	2	---	---	2	---	---	1	0	0
KBBKQ	3-2	0	1	0	0	1	0	---	5	---	---	17	---	0	1	0	---	17	---	---	5	---	0	1	0
KBBKR	3-2	3	0	0	3	0	0	1	---	---	19	---	---	1	0	0	19	---	---	1	---	---	1	0	0
KBKP	2-2	0	1	0	0	1	0	---	1	---	---	12	---	0	1	0	---	12	---	---	1	---	0	1	0
KBNKB	3-2	45	0	0	1	0	0	10	---	---	36	---	---	1	0	0	36	---	---	10	---	---	1	0	0
KBNKN	3-2	922	0	0	1	0	0	67	---	---	97	---	---	1	0	0	97	---	---	67	---	---	1	0	0
KBNKP	3-2	61	1	0	1	1	0	13	1	---	36	45	---	4	1	0	37	45	---	8	1	---	0	1	0
KBNKQ	3-2	0	1	0	0	1	0	---	33	---	---	44	---	0	1	0	---	44	---	---	33	---	0	1	0
KBNKR	3-2	6	2	0	1	1	0	5	8	---	26	28	---	1	1	0	32	28	---	1	8	---	0	1	0
KBPK	3-1	6	0	0	1	0	0	14	---	---	20	---	---	1	0	0	20	---	---	14	---	---	1	0	0
KBPKB	3-2	160	0	0	2	0	0	33	---	---	44	---	---	2	0	0	44	---	---	33	---	---	2	0	0
KBPKN	3-2	2112	13	0	2	12	0	37	2	---	47	2	---	2	12	0	50	2	---	19	2	---	0	12	0
KBPKP	3-2	403	2	1	1	1	1	20	2	2+8	31	11	16+22	1	1	1	31	14	16+22	20	1	2+8	1	0	1
KBPKQ	3-2	0	16	0	0	2	0	---	20	---	---	28	---	0	1	0	---	28	---	---	20	---	0	1	0
KBPKR	3-2	4	302	0	1	1	0	3	15	---	32	29	---	1	1	0	32	33	---	3	14	---	1	0	0
KBPPK	4-1	6	0	0	1	0	0	6	---	---	12	---	---	1	0	0	16	---	---	1	---	---	0	0	0
KNKP	2-2	22	7	0	6	1	0	6	9	---	6	24	---	6	1	0	6	24	---	6	9	---	6	1	0
KNNKN	3-2	362	0	0	85	0	0	4	---	---	4	---	---	85	0	0	4	---	---	4	---	---	85	0	0
KNNKP	3-2	3124	19	0	1	2	0	105	3	---	105	39	---	1	1	0	105	53	---	105	1	---	1	0	0
KNNKQ	3-2	0	229	0	0	1	0	---	53	---	---	62	---	0	1	0	---	62	---	---	53	---	0	1	0
KNNKR	3-2	0	25	0	0	1	0	---	5	---	---	28	---	0	1	0	---	35	---	---	2	---	0	0	0
KNPK	3-1	75	0	0	1	0	0	14	---	---	19	---	---	2	0	0	19	---	---	14	---	---	1	0	0
KNPKB	3-2	640	2	0	1	1	0	28	2	---	40	2	---	1	1	0	40	2	---	28	2	---	1	1	0
KNPKN	3-2	4128	63	0	3	42	0	40	2	---	56	2	---	1	42	0	56	2	---	40	2	---	1	42	0
KNPKP	3-2	2281	14	8	1	6	1	21	5	1+13	29	24	17+28	1	1	1	33	24	17+28	19	5	1+13	0	1	1
KNPKQ	3-2	0	52	0	0	1	0	---	29	---	---	45	---	0	1	0	---	45	---	---	29	---	0	1	0
KNPKR	3-2	23	1158	0	2	1	0	4	34	---	19	58	---	1	1	0	28	58	---	3	34	---	0	1	0
KNPPK	4-1	93	0	0	6	0	0	5	---	---	11	---	---	1	0	0	16	---	---	1	---	---	0	0	0
KPK	2-1	80	0	0	1	0	0	18	---	---	27	---	---	1	0	0	27	---	---	18	---	---	1	0	0
KPKP	2-2	106	106	15	1	1	15	10	10	1+1	24	24	19+12	2	2	1	27	27	19+12	6	6	1+1	0	0	1
KPPK	3-1	43	0	0	6	0	0	11	---	---	26	---	---	1	0	0	26	---	---	11	---	---	1	0	0
KPPKB	3-2	211	1	0	2	1	0	14	2	---	25	2	---	1	1	0	28	2	---	12	2	---	0	1	0
KPPKN	3-2	920	157	0	1	11	0	22	12	---	36	12	---	2	15	0	36	12	---	22	12	---	1	11	0
KPPKP	3-2	4179	52	6	3	2	1	24	7	6+3	38	18	15+19	1	2	1	86	23	16+20	10	1	4+3	0	0	0
KPPKQ	3-2	0	2	0	0	1	0	---	8	---	---	13	---	0	1	0	---	13	---	---	8	---	0	1	0
KPPKR	3-2	18	99	2	1	1	1	11	21	1+4	39	36	18+17	1	1	1	39	36	15+21	11	21	3+1	1	1	0
KPPPK	4-1	11	0	0	2	0	0	7	---	---	17	---	---	1	0	0	17	---	---	7	---	---	1	0	0
KQBKQ	3-2	25	0	0	1	0	0	9	---	---	11	---	---	1	0	0	14	---	---	6	---	---	0	0	0
KQNKQ	3-2	38	0	0	1	0	0	24	---	---	30	---	---	1	0	0	30	---	---	24	---	---	1	0	0
KQPKQ	3-2	640	0	0	1	0	0	91	---	---	102	---	---	1	0	0	102	---	---	91	---	---	1	0	0
KQPKR	3-2	1	0	0	1	0	0	2	---	---	11	---	---	1	0	0	11	---	---	2	---	---	1	0	0
KQRKQ	3-2	1	0	0	1	0	0	11	---	---	18	---	---	1	0	0	18	---	---	11	---	---	1	0	0
KRBKP	3-2	1	0	0	1	0	0	1	---	---	8	---	---	1	0	0	8	---	---	1	---	---	1	0	0
KRBKQ	3-2	0	372	0	0	1	0	---	38	---	---	67	---	0	1	0	---	67	---	---	38	---	0	1	0
KRBKR	3-2	17	0	0	1	0	0	49	---	---	55	---	---	1	0	0	55	---	---	49	---	---	1	0	0
KRKB	2-2	5	0	0	1	0	0	14	---	---	25	---	---	1	0	0	25	---	---	14	---	---	1	0	0
KRKN	2-2	18	0	0	2	0	0	10	---	---	24	---	---	2	0	0	24	---	---	10	---	---	2	0	0
KRKP	2-2	12	0	0	6	0	0	6	---	---	20	---	---	4	0	0	20	---	---	6	---	---	4	0	0
KRNKN	3-2	3	0	0	1	0	0	19	---	---	24	---	---	2	0	0	24	---	---	19	---	---	1	0	0
KRNKQ	3-2	0	455	0	0	1	0	---	42	---	---	63	---	0	1	0	---	63	---	---	42	---	0	1	0
KRNKR	3-2	10	0	0	2	0	0	22	---	---	23	---	---	2	0	0	23	---	---	22	---	---	2	0	0
KRPKB	3-2	225	0	0	1	0	0	58	---	---	67	---	---	1	0	0	67	---	---	58	---	---	1	0	0
KRPKN	3-2	413	0	0	4	0	0	37	---	---	47	---	---	1	0	0	47	---	---	37	---	---	1	0	0
KRPKP	3-2	0	2	1	0	2	1	---	1	1+4	---	15	15+15	0	1	1	---	15	15+15	---	1	1+4	0	1	1
KRPKQ	3-2	2	241	0	2	1	0	2	63	---	12	91	---	1	1	0	12	91	---	2	63	---	1	1	0
KRPKR	3-2	209	0	0	1	0	0	43	---	---	54	---	---	2	0	0	54	---	---	43	---	---	1	0	0
KRRKB	3-2	1	0	0	1	0	0	8	---	---	16	---	---	1	0	0	16	---	---	8	---	---	1	0	0
KRRKQ	3-2	10	0	0	1	0	0	7	---	---	20	---	---	1	0	0	20	---	---	7	---	---	1	0	0

Table 1. Mutual zugzwangs: the statistics.

As is usual, the statistics of Table 1 may be affected by the presence of unreachable positions in which the side-not-to-move is not in check. Elkies (2000) and van der Heijden (2000) have both pointed out such positions:

KBPKP: 8/8/8/8/1p6/1k6/1P6/BK6
KBPPK: kB6/8/1PK5/1P6/8/8/8/8, kB6/8/KP6/1P6/8/8/8/8 ... leaving 4 of the 6
KNPPK: kN6/8/KP6/1P6/8/8/8/8, kN6/8/1PK5/1P6/8/8/8/8, K1k5/P1PN4/8/8/8/8/8 ... leaving 90 of 93.

Endgame		type	dc	dm	a maximal mzug
KBBKN	0023	cm ww	2	2	8/8/8/8/6n1/2K4B/kB6
KBBKP	0020.01	cm ww	2	2	B1k5/1pB5/3K4/8/8/8/8/8
KBBKQ	3020	cm bw	5	17	8/8/8/8/q7/2BB4/1K6/3k4
KBBKR	0320	cm ww	1	19	8/8/8/B7/8/3k4/2r5/KB6
KBKP	0010.01	cm bw	1	12	8/8/8/8/8/8/1pK5/kB6
KBNKB	0041	cm ww	10	36	8/8/8/8/1b6/8/2K5/k1BN4
KBNKN	0014	cm ww	67	97	8/8/2B2n2/8/N7/8/3K4/1k6
KBNKP	0011.01	c ww	13	36	8/8/2K5/8/2k5/5p2/8/2N1B3
		m ww	8	37	8/8/8/8/8/B7/p3N3/k2K4
		cm bw	1	45	8/8/8/1N6/3K4/B7/5p2/k7
KBNKQ	3011	cm bw	33	44	8/8/q7/8/3K4/2N5/8/k1B5
KBNKR	0311	c ww	5	26	3r4/8/2B5/8/1N6/8/8/k1K5
		m ww	1	32	8/8/8/8/B7/1r6/N1k5/K7
		cm bw	8	28	8/r7/8/B7/8/8/N1k5/K7
KBPK	0010.10	cm ww	14	20	1B1K4/8/8/k7/8/P7/8/8
KBPKB	0040.10	cm ww	33	44	8/8/8/2P5/4b3/1B6/8/k1K5
KBPKN	0013.10	c ww	37	47	8/8/8/8/8/K5n1/B5P1/k7
		m ww	19	50	k7/8/Kn1BP3/8/8/8/8/8
		cm bw	2	2	K7/P1k5/8/8/8/1B6/1n6/8
KBPKP	0010.11	cm ww	20	31	2K5/8/3k4/8/7p/5P2/8/7B
		c bw	2	11	8/8/8/8/8/1k6/1P1p4/KB6
		m bw	1	14	8/8/8/8/8/2p5/2P5/kBK5
		cm fp	2+8	16+22	8/8/8/8/8/k1p5/2P5/1BK5
KBPKQ	3010.10	cm bw	20	28	5k2/1P1K4/1qB5/8/8/8/8/8
KBPKR	0310.10	cm ww	3	32	K7/1rB1P3/k7/8/8/8/8/8
		c bw	15	29	8/8/4r3/8/2k5/K6P/5B2/8
		m bw	14	33	8/8/8/8/k7/r1P5/1K6/B7
KBPPK	0010.20	c ww	6	12	8/B1k5/K7/P7/P7/8/8/8
		m ww	1	16	kB6/8/1PK5/1P6/8/8/8/8
KNKP	0001.01	cm ww	6	6	8/8/8/8/8/p7/k2N4/2K5
		cm bw	9	24	8/K7/N1k5/p7/8/8/8/8
KNNKN	0005	cm ww	4	4	8/8/8/8/n7/8/2KN4/kN6
KNNKP	0002.01	cm ww	105	105	8/8/8/p7/K7/4k3/8/6NN
		c bw	3	39	7N/8/K1N5/8/8/1pk5/8/8
		m bw	1	53	1N6/8/8/N7/K7/8/kp6/8
KNNKQ	3002	cm bw	53	62	8/8/1q6/8/4N3/3K2N1/8/4k3
KNNKR	0302	c bw	5	28	6rN/5N2/8/8/8/2k5/8/1K6
		m bw	2	35	5N2/1N6/8/3r4/8/2k5/8/2K5
KNPK	0001.10	cm ww	14	19	8/8/8/8/8/1k1P4/8/KN6
KNPKB	0031.10	cm ww	28	40	8/6b1/8/8/1N5P/8/2K5/k7
		cm bw	2	2	K5b1/P1k2N2/8/8/8/8/8/8
KNPKN	0004.10	cm ww	40	56	2n5/8/8/1N6/8/6P1/8/k1K5
		cm bw	2	2	K7/P2n4/1k6/N7/8/8/8/8
KNPKP	0001.11	c ww	21	29	8/8/8/8/8/2k2p2/P7/1KN5
		m ww	19	33	K7/N6p/k7/8/8/8/7P/8
		cm bw	5	24	8/8/8/8/6P1/3k2p1/8/2KN4
		cm fp	1+13	17+28	8/8/8/8/8/1k2p3/4P3/KN6
KNPKQ	3001.10	cm bw	29	45	4N3/3P4/2K5/q7/1k6/8/8/8
KNPKR	0301.10	c ww	4	19	k6r/5P2/K7/3N4/8/8/8/8
		m ww	3	28	3k4/r1N5/2KP4/8/8/8/8/8
		cm bw	34	58	8/8/8/5N2/K7/2k5/P2r4/8
KNPPK	0001.20	c ww	5	11	K7/P1k5/P7/8/8/8/8/N7
		m ww	1	16	kN6/8/1PK5/1P6/8/8/8/8
KPPK	0000.10	cm ww	18	27	8/8/8/1k6/8/1K6/6P1/8
KPPKP	0000.11	c ww	10	24	8/k7/6p1/K5P1/8/8/8/8
		m ww	6	27	8/8/k7/8/K5p1/8/5P2/8
		c bw	10	24	8/8/8/8/k5p1/6P1/K7/8
		m bw	6	27	8/5p2/8/k5P1/8/K7/8/8
		cm fp	1+1	19+12	8/1pK5/kP6/8/8/8/8/8
KPPPK	0000.20	cm ww	11	26	8/8/8/8/1k6/1P6/1PK5/8
KPPPKB	0030.20	c ww	14	25	1b6/8/3P4/4P3/8/8/8/3K1k2
		m ww	12	28	8/8/5b1P/8/1K6/1P6/1k6/8
		cm bw	2	2	8/8/8/8/8/b2k4/P2P4/1K6
KPPKN	0003.20	cm ww	22	36	8/5n2/8/K6P/3P4/k7/8/8
		cm bw	12	12	K7/2k5/8/P7/P7/5n2/8/8
KPPKP	0000.21	c ww	24	38	1K3k2/8/7p/8/8/7P/6P1/8
		m ww	10	86	8/2p5/8/8/8/2k1P3/P7/3K4
		c bw	7	18	K1k5/8/7p/P7/2P5/8/8/8
		m bw	1	23	8/2p5/8/8/1P1P4/2k5/8/2K5
		c fp	6+3	15+19	8/8/8/2k5/K1p5/P3P3/8/8
		m fp	4+3	16+20	8/8/8/8/5k2/3K1p2/3P3P/8
KPPKQ	3000.20	cm bw	8	13	8/2KP3q/8/2P3k1/8/8/8/8
KPPKR	0300.20	cm ww	11	39	2k5/K6P/6Pr/8/8/8/8/8
		cm bw	21	36	8/8/8/8/2P5/2K1P3/4r3/2k5
		c fp	1+4	18+17	1r1k4/1P6/1PK5/8/8/8/8/8
		m fp	3+1	15+21	8/8/8/8/k7/r1P5/1KP5/8
KPPPK	0000.30	cm ww	7	17	8/8/8/1k1P4/8/PK6/P7/8
KQBKQ	4010	c ww	9	11	8/3K4/3B4/8/k7/3Q4/8/2q5
		m ww	6	14	1q6/8/2Q5/B7/8/1k6/8/1K6
KQNKQ	4001	cm ww	24	30	8/3q4/1Q1N4/8/k7/8/3K4/8
KQPKQ	4000.10	cm ww	91	102	8/8/8/8/2K3Q1/2P1q3/8/4k3
KQPKR	1300.10	cm ww	2	11	k7/8/KQ1r4/P7/8/8/8/8
KQRKQ	4100	cm ww	11	18	8/8/8/8/1R6/k4q2/8/1K2Q3
KRBKP	0110.01	cm ww	1	8	1k1K4/7R/8/8/8/8/6p1/7B
KRBKQ	3110	cm bw	38	67	1q6/8/1B3R2/8/k7/8/8/1K6
KRBKR	0410	cm ww	49	55	5R2/8/8/8/8/3K4/5Br1/2k5
KRKB	0130	cm ww	14	25	8/8/1b6/5R2/8/3K4/8/2k5
KRKN	0103	cm ww	10	24	8/8/8/6n1/3K4/4R3/3k4/8
KRKP	0100.01	cm ww	6	20	8/K7/8/k7/1p6/8/8/1R6
KRNKN	0104	cm ww	19	24	8/8/8/8/8/3n4/N2k4/RK6
KRNKQ	3101	cm bw	42	63	1Nk5/8/8/8/8/1R6/6q1/2K5
KRNKR	0401	cm ww	22	23	8/8/8/8/8/2KRN3/8/2k1r3
KRPKB	0130.10	cm ww	58	67	8/8/8/8/7R/1k2P3/2b5/K7
KRPKN	0103.10	cm ww	37	47	K7/1R6/2n5/4k3/8/4P3/8/8
KRPKP	0100.11	cm bw	1	15	8/8/8/8/8/1p6/kP6/1RK5
		cm fp	1+4	15+15	8/8/8/8/8/2p5/1kP5/2RK4
KRPKQ	3100.10	cm ww	2	12	7k/2K2P1q/8/8/8/8/5R2/8
		cm bw	63	91	8/8/q1k5/8/1R1K4/8/1P6/8
KRPKR	0400.10	cm ww	43	54	8/8/8/8/8/2RPr3/8/2K1k3
KRRKB	0230	cm ww	8	16	8/8/8/8/8/b1k5/1R6/1RK5
KRRKQ	3200	cm ww	7	20	6R1/8/8/8/6R1/7q/1K5k/8

Table 2. Maximal mutual zugs: examples.

For a given endgame and type of mzug, the sets *ZC* and *ZM* are the sets of DTC-maximal and DTM-maximal mzugs respectively. ZC & ZM are either disjoint, identical or one is a subset of the other. There is no endgame for which ZC - ZM $\neq \varnothing$ and ZM - ZC $\neq \varnothing$.

ZC ∩ ZM = ∅	KBNKP ww, KBNKR ww, KBPKN ww, KBPKP bw, KBPKR bw, KNNKP bw, KNNKR bw, KNPKP ww, KNPKR ww, KPKP ww (and therefore bw), KPPKB ww, KPPKP ww, bw and fp, KPPKR fp and KQBKQ ww.
ZC ⊂ ZM	KNPK ww (1-2), KPPKN ww (1-2) and bw (11-15), KRNKN ww (1-2) and KRPKR ww (1-2)
ZC ⊃ ZM	KBBKR ww (3-1), KBPKQ bw (2-1), KNPKN ww (3-1), KNPKP bw (6-1), KPKP fp (15-1), KPPK ww (6-1), KRKP ww (6-4), KRPKN ww (4-1), KRPKP bw (2-1), KRPKQ ww (2-1)

4 SUMMARY

A complete set of data about 3-to-5-man mzugs in chess has been compiled, second-sourced, summarised here and made available via the web. This includes maximal mzug and depth statistics to both the DTC and DTM metrics.

5 ACKNOWLEDGEMENTS

Thanks go first to Eugene Nalimov, Peter Karrer and Christoph Wirth for generating and scanning complete suites of 3-5-man EGTs. Thanks also go to Thomas Lincke and John Tamplin for their web interfaces to the EGTs, to Lars Rasmussen for his published and unpublished corpus of mzug information, and to John Roycroft for access to past copies of EG.

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